Anchoring Theory — Compuserve Sailing Forum Discussion
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We are all aware that real-world anchoring is more complex than an idealized mathematical model or lab test. Otherwise we wouldn't see such wide variances among anchor tests. Nor would there be so much for sailors to argue about! With that caveat in this little writeup I will focus mainly on the relatively simple problem of determining what weight of various spade anchor designs is suitable for a given vessel's wind drag load. Other critical issues such as the probability of setting or resetting in various unfriendly bottom-types are vigorously ignored.

My interest in the theoretical treatment of ground tackle loads and anchor performance is that it may help us understand:

a) What power-laws are we dealing with? E.g., if the weight of a sediment-cleaving anchor necessary to resist a given wind velocity load is a function of the cube of wind velocity that's something we all need to understand.

b) How can we normalize the holding power of different weights of different designs. E.g., the tests in reference R3 showed a 35 lb. Delta generated less average resistance (801 lb.) than a 47 lb. CQR (1304 lb.). Is that what we should expect from the relative weights? Then how would heavy should the Delta design be to equate to the CQR? From the R3 test report it's not even clear whether it should be less or more than the CQR.

The references used herein are:

R1 - The Complete Book Of Anchoring And Mooring, Earl Hinz 1986
R2 - Oceanography And Seamanship, Second Edition, William G. Van Dorn 1993
R3 - The Sailing Foundation Anchor Tests Puget Sound, 1995, Doug Fryer
R4 - West Marine Advisor - Anchors 1996 Pg 143 ABYC Ground Tackle Loads Table
R5 - Anchors Selection And Use, Robert A. Smith 1996

I've attempted to organize the information into the following sections:

I. Van Dorn Method
II. Normalization of Anchor Tests
III. Ground Tackle Loads - Wind Drag
IV. Predicted/actual Anchor Performance

I. Van Dorn Method

Since Van Dorn's 2nd edition was published I've been most intrigued with his efforts to quantify the problem analytically, rather than empirically with various "fudge factor" adjustments. Among the assertions that particularly caught my attention were:

- the complex dynamic problem becomes statics given sufficient rode elasticity
- anchor weight required is a function of cube of wind velocity
- anchor performance is proportional to anchor weight^2/3
- pitching and surging don't really impact success (at least until you can significantly reduce yaw/sway swinging)

In my view Van Dorn has made a major contribution to our safety at sea with his presentation of the physics of anchoring. So I have attempted in the following to outline his methodology for the benefit of those members who have not (yet) acquired this valuable reference, incorporating contributions from R1 and R5 where I found them useful.

Following are some of the answers I need to know for our own new boat:

1. Realistic wind drag load as a function of (velocity, yaw-angle).

2. Relative anchor performance in good-holding bottoms as a function of anchor (size, design).

3. Proportion of nylon rode required for sufficient elasticity.

4. How to connect the length of shock-absorbing nylon to the chain so it stays connected, particularly when storm-anchored.

Van Dorn's anchoring system design assumptions are:

- Maximum 8 deg stock angle, approximated by maximum 8° rode angle at the bow
- Maximum stress on nylon of 25% of breaking (i.e., 1/2 of non-deforming stretch). Note - ABYC assumes a maximum of 12%.

Van Dorn's analysis decomposes into the following steps:

A. Anchoring dynamics
B. Static load equilibrium
C. Minimum safe anchoring depth
D. Wind force/rode selection
E. Anchor holding force and selection

A. Anchoring dynamics

Van Dorn begins with the compelling point that because anchoring dynamics are so complex, most authors resort to a static analysis multiplied by huge safety factors. Besides possibly leading sailors to carry and handle much heavier gear than appropriate, even the "oversized" gear sometimes doesn't work.

In brief, Van Dorn agrees that the peak load is yaw-sway (swinging), but eliminates consideration of the impulse loads by the rode elasticity — thereby reducing the problem to computation of static wind load at a yaw angle of 30 degrees.

The elastic restraint of the rode and anchor converts the vessel's 3 non-oscillatory modes of motion (surge, sway, yaw) into 6 oscillatory modes. Roll isn't significantly effected by anchoring restraint, the remaining modes are grouped into:

pitch/heave	- of no effect on anchor holding power
surge	- only resonant if tightly moored at dockside, much smaller forces than sway/yaw
sway/yaw	- the primary cause of anchor-system failure

He concludes that only elastic anchoring is suitable for storm anchoring, and we should seriously strive to minimize swinging.

B. Static load equilibrium

Van Dorn says "The object of this section is to show how best to proportion these elements for optimum versatility: as we shall see, it is perfectly feasible to design an anchoring system adequate for everyday use that can also be configured for storm anchoring." I'm sure every cruiser coming through the Bay of Islands would very keen to understand how to accomplish this.

He then reduces the equilibrium equations to this simple approximation, asserting that it will achieve his assumption of no more than 8° anchor stock angle:

Eq 1: lengthNylon + 0.8*lengthChain = 4.8*waterDepth pg 388

That implies a scope of 4.8 for an all-nylon rode, or 6.0 all-chain (I'm not sure why Van Dorn states 7.2, p 389 — most likely an editorial slip.)

Whether additional scope beyond that required by Eq 1 will further reduce stock angle and enhance holding resistance is not addressed here. Earl Hinz, Pg 97 Fig 5-1, includes a curve relating relative holding power to scope. It shows that in sand an improvement of about 100% as rode lead angle is reduced from 8° to 0°. The reference for this relationship is not given.

From here Van Dorn assumes you have sufficient nylon, regarding the chain as an abrasion-resistant connection to the anchor. There is no further discussion of the magnitudes of the impulses that must be absorbed, other than that a 30° yaw increases wind drag about 300%.

We need to know how to determine how much nylon is "enough". In particular, we would prefer to anchor in common shallow water coral sites first on all-chain, adding a nylon "snubber" shock absorber of appropriate length connected to our bow bridle system.

It seems clear to me that we are transfering energy from the impulse to the rode, mostly potential energy until the loads induce permanent deformation and associated heat. Nylon stretches roughly 0.2 and 0.3 ft/ft at 11% and 30% respectively of breaking. I have stumbled at this point as I do not understand how to translate the stretch vs. load relationship into the required amount of impulse absorption.

A detail, but for completeness the waterDepth used above should also incorporate maximum probable wave heights as well as tide effects.

C. Minimum safe anchoring depth

Here Van Dorn applies his elegant Cumulative Sea State ("CSS") diagram to derive maximum probable wave heights, given fetch and windSpeed, to accomodate surge and heave motions. He doesn't appear to be concerned about the forces, but to ensure the vessel remains well clear of bottom. This seems to me to be an absolutely critical part of setting up a storm anchoring system appropriate to a particular site — but nowhere other than Van Dorn have I seen a solid treatment incorporating wave effects.

D. Wind force/rode selection

Here Van Dorn discusses how to predict the maximum wind and current forces. Allowing 10% for current, an extra 15% for powerboats over sail/bare poles, and 30° swinging he introduces the wonderfully simple formula, where DSPL is in long tons:

Eq 2: rodeForce = 0.20 * windSpeed^2 * DSPL^(2/3) (lbs) pg 392

Taking our DSPL very conservatively as 30,000 lbs., wind = 60kn, we get rodeForce = 4,061 lb.. The ABYC table for 50' boat gives 6,400 lb.. By applying the Earl Hinz method to our actual areas I derived 2,642 lb. (without applying his “surge factor”). I wonder which, if any, is correct (and I hope ABYC is wrong).

In section III below I'll return to this load calculation with the details.

E. Anchor holding force

Van Dorn, quite correctly in my view, considers only spades for storm anchoring, as hooks only depend on being strong enough not to break under rode tension. Van Dorn states "anchor holding force... is well demonstrated to be proportional to fluke area (i.e., to the two-thirds-power of its weight...":

Eq 3: rodeForce = sedimentCoeff * anchorCoeff * anchorWeight ^ (2/3). Pg 392

That holding force is proportional to fluke area, i.e., anchorWeight^2/3, seems intuitive if one assumes that the thickness of all the anchor sections increases in proportion to the increase in fluke area. Two other authors offering confirmations of the anchor weight^2/3 power law are:

- Earl Hinz in Fig 6-5 shows a linear regression line on Simpson Lawrence CQR tests F = K*W^2/3. The relationship is not otherwise addressed in the text.

- Robert A. Smith in R5 also makes this assertion as "For anchors of the same type and of proportionate dimensions but of different size all areas, including the stressed area of soil, will be proportionate to the squares of any linear dimensions...The volumes, and therefore the weights of steel anchors, will vary as the cubes of the same linear dimensions. If follows that the stressed areas of soil, and consequently the holding powers, are in proportion to the two thirds powers (the squares of the cube roots) of the weights." Smith then shows Figure 1 —a linear regression of Danforth HT tests as evidence of good fit for 5 data points.

To relate anchor weight to the boat and windspeed, combining Eq 2 and Eq 3 he obtains

Eq 4: anchorWeight = DSPL * windSpeed^3 / Ω / (sedimentCoeff * anchorCoeff)^1.5 Pg 392

where Van Dorn gives Ω = 10. Doing the algebra I obtained Ω = 11.18. I cross-checked my results against his Fig 156 and by back-calculation determined that this chart appears to be based upon a Ω value of 45. In my calculations I used the value of 11.18.

Regardless of what is the correct value of the proportionality coefficient, given that anchor holding is proportional to weight^2/3, then it is clearly correct that anchor weight must be proportional to velocity^3, a critically important point.

There are a couple of other minor errors in the text that I think I found:

- The single "anchor coefficient" Cs should be Ca, Pg 392.

- Before proceeding, I verified my rodeForce calculations using Eq 2 against his Fig 155. My results are some 10 to 15% higher than the Van Dorn chart on the samples I checked for the 10 long ton DSPL case.

- Pg 393, example 2, Van Dorn states - if anchor is CQR it must weigh 40 lb., whereas I obtain 96 lb..

II. Normalization of Anchor Tests

When I first read the excerpts from the Puget Sound test (R3) I thought "this is a great test, but how can we force rank spade-type anchors of such different weights?" Some have quoted this test as proof that the Simpson Lawrence Delta design is inferior to the classic CQR. To me that is misleading — that a 35 pound Delta drags at a lower force than a 47 pound CQR isn't surprising. What is much more useful to know is what weight Delta or Bruce anchors would perform the same as the 47 pound CQR. In the following I attempt to address that question.

I recently received courtesy of Chuck Hawley the full text of the Puget Sound report, including the raw test data and summaries of several other anchor tests. The following analysis is based upon a sample comprised of:

- the anchor tests reported by Earl Hinz
- the Puget Sound tests
- all the other tests summarized in R3, such as Florida and San Francisco tests

The purpose of the following model is to determine the set of sediment (site) and anchor coefficients which satisfies the system of equations formed by applying Equation 3 to this data set. Please note that this system of equations is underdetermined with respect to the two sets of coefficients. That is, the pair of coefficient sets which satisfy the system are defined in terms of each other. If you think of two vectors of coefficients {s} and {a} for sites, anchors, respectively, which satisfy the system, then for any factor F the vectors F{s} and {a}/F also satisfy the system. In the following iterative solution I used initial values for {s} and {a} similar to Van Dorn's suggested coefficients so that the resulting solution set has a similar range of values.

I wrote an Excel spreadsheet model to solve the system of equations by simple iteration, terminating the iteration when the average values of **predicted performance** divided by **actual performance** for:

- each anchor design averaged over all sites (see Table B for coefficient set {a})
- each site averaged over all anchors (see Table A for coefficient set {s})

are 1.0 (i.e., 100%) and none of the {a} or {s} coefficients changed by more than 0.001 from the previous iteration.

Table A — Site Coefficients & Statistics

Test	Site	Sediment Coeff	Num Samples	Average Pred/Actual	Std Deviation Pred/Actual
Hinz data	A	6.6	2	100%	14%
Hinz data	B	2.9	2	100%	57%
Hinz data	C	3.1	2	100%	21%
Hinz data	D	10.0	2	100%	10%
Dutch test	DT	9.2	4	100%	32%
Hinz data	E	7.3	2	100%	35%
Florida	FL	19.0	3	100%	6%
French test	FR1	10.8	2	100%	1%
French test	FR2	10.9	2	100%	71%
French test	FR3	16.0	2	100%	37%
Max tests	M1	1.7	3	100%	57%
Max tests	M2	3.2	3	100%	50%
Puget Sound	PS1	6.1	77	100%	59%
Puget Sound	PS2	4.7	11	100%	63%
Puget Sound	PS3	4.9	28	100%	58%
Puget Sound	PS4	2.9	7	100%	54%
Puget Sound	PS5	5.8	21	100%	35%
San Francisco	SF	2.9	5	100%	37%

To get an idea of what the difference in anchor coefficients really means, I also computed the equivalent anchor weights and +/- two sigma (standard deviation) bounds estimates. I chose as the reference an 88 lb. Delta as it's the anchor we are considering for our general purpose anchor. The effect of the 2/3 power law may surprise some readers — look at the extremes of the Bruce and Fortress. Following are the resulting anchor coefficients and equivalent weights where weights/forces are in pounds:

Table B — Anchor Coefficients & Statistics

Anchor	Anchor Coeff	Num Samples	Average Pred/Actual	Standard Deviation	Equiv Weights	Holding -2 sigma	Average Holding	Holding +2 sigma
Bruce	6.6	45	100%	39%	220	305	1,446	2,587
CQR	12.7	44	100%	50%	83	5	1,446	2,888
Delta	12.2	35	100%	44%	88	169	1,446	2,724
Fortress	51.1	17	100%	78%	10	0	1,446	3,716
Max	13.0	20	100%	74%	80	0	1,446	3,577
WMPerf	21.0	17	100%	43%	39	209	1,446	2,684

Note that the Van Dorn model predicts an average holding power of 1,446 lbs. for this 88lb. anchor in a bottom with a site coefficient of 6. The weights of the other anchor designs were then back-calculated to match the 1,446 lb. holding force. The sigmas are of course biased towards the

Puget Sound test data, which is about 80% of the sample I have today. The zeros of holding power at minus two sigmas results from my irritation with seeing the negative numbers which result from such high sigmas.

As Chuck Hawley has pointed out many times, the variability of the results is impressive — even though the testers tried very hard to design repeatable tests. The Dutch tested literally in a sandbox - it would be very interesting to see what the interval variability of their tests were!

The overall sample is still tiny given the obvious variability of pull testing. Nevertheless, it's leading me to some tentative conclusions:

1. Three anchors - CQR, Delta, Max seem to perform pretty close to each other on a similar weight basis. Note there are relatively few Max samples.

2. The pull test variability expressed as standard deviation appears not to be a differentiator among anchor designs, except possibly for the Fortress and Max. Even for those 2 anchors the statistics may be distorted by the inability of the test rigs (or methods) to handle extremely high holding powers.

3. The wide range of sediment coefficients probably also reflects differences in test method (in addition to real world differences in bottom holding characteristics) - the French and Florida tests look possibly like outliers.

III. Ground Tackle Loads - Wind Drag

The purpose of this section is to compare the three sources I have on how to estimate the wind drag of a given boat. Unfortunately I obtained three very different results. Van Dorn gives a formal method for computing wind drag from projected areas on pg 276. I didn't attempt that method simply because I stumbled in my attempt to derive this formula from first principles. Instead in the following comparitive tabulation I have applied Van Dorn's elegant displacement-based estimator:

Eq 5: rodeForce = 0.20 * windSpeed^2 * DSPL^(2/3) (lbs) pg 392

The Earl Hinz method for computing wind drag from characteristic area appeared straightforward, so I measured the our abuilding 52-ft catamaran athwartships (F) and lateral (L) areas using Autocad, including all spars and rigging. I recall reading that yacht rigging wind drag is larger than would be implied by using the actual dimensions of wires or rod. But Hinz doesn't allow for this, and I have no other reference to guide me. To approximate a worst case 30° yaw, I computed the characteristic projected area as:

Eq 6: characteristicArea = F * cos30 + L * sin30 = 395 sq.ft. (my trig, not Hinz)

Per R1 pg 20, Hinz gives:

Eq 7: wind drag = q * dragCoeff * characteristicArea,

where q = V^2 * rho/2. Hinz gives rho=.002378, whereas his table on the same page requires rho=0.0068 which is what I used (unfortunately I have no other reference to crosscheck these physical constants).

I used dragCoeff = .55 for our new 52' catamaran Adagio as per the Hinz Table 2-3 pg 21. I did not apply his Surge Factor because it appears inappropriate as I believe Van Dorn is correct that sufficient rode elasticity reduces this to a statics problem.

Following is the resulting table of wind drag by the Hinz and VanDorn methods as compared to the ABYC anchoring load table per R4 West Marine Advisor for a 50' boat (forces in pounds):

Wind	**q**	**Wind Force**	**Wind Force**	**Wind Force**

Velocity		Hinz	Van Dorn	ABYC-50'
15	0.76	165	194	400
30	3.04	661	775	1,600
42	5.95	1,295	1,518	3,200
60	12.15	2,642	3,099	6,400

The wind loads by the Van Dorn displacement method were computed using a conservative displacement of 30,000 lbs. A DSPL of 60,000 lb. would increase the estimated drag to agree with the ABYC table. I need some help here from someone who knows their aerodynamics much better than I!

IV. Predicted/actual Anchor Performance

Please see attached spreadsheet for the test data details.